Запутанные состояния, коты Шредингера и нелокальность в квантовой механике

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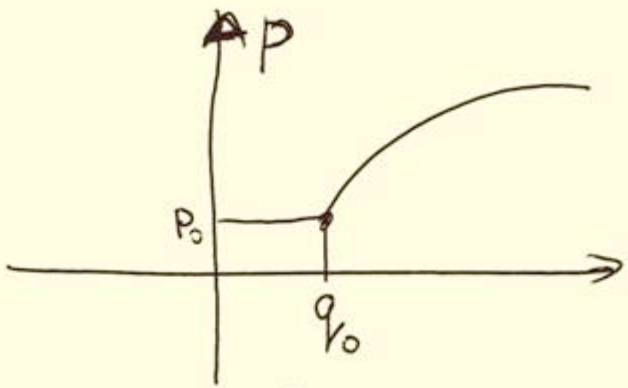
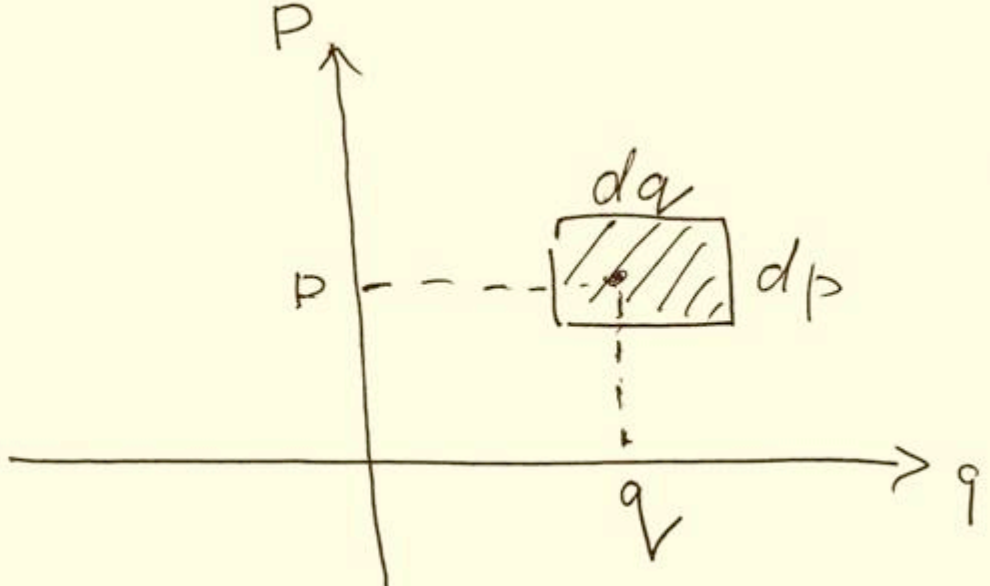
$q \qquad p = \dot{q}$



$q(t) \quad p(t)$

$$H = \frac{p^2}{2} + U(q)$$

$$\dot{p} = -\frac{\partial U(q)}{\partial q}$$



$$f(q,p) \geqslant 0 \; ; \quad \int f(q,p)\,dq\,dp = 1$$

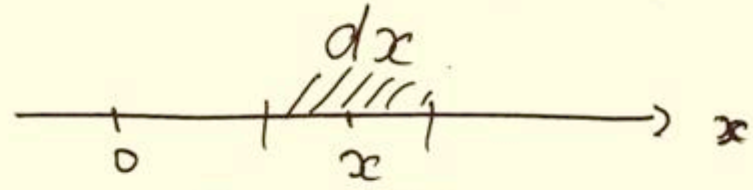
$$\int f(q,p)\,dp = P(q);$$

$$\int f(q,p)\,dq = \Pi(p).$$

$$\frac{\partial f(q,p,t)}{\partial t} + p\,\frac{\partial f(q,p,t)}{\partial q} - \frac{\partial U(q)}{\partial q}\,\frac{\partial f(q,p,t)}{\partial p} = 0$$

(1926)

$$\Psi(x) = |\Psi(x)| \exp i\Phi(x) = \langle x|\Psi\rangle$$



$$|\Psi(x)|^2 = P(x)$$

$$\hat{H} = -\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x)$$

$$i\frac{\partial \Psi(x,t)}{\partial t} = -\frac{1}{2}\frac{\partial^2}{\partial x^2}\Psi(x,t) + V(x)\Psi(x,t)$$

$$i\frac{\partial}{\partial t}|\Psi,t\rangle = \hat{H}|\Psi,t\rangle$$

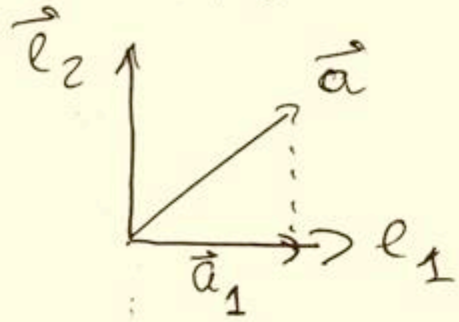
(1927) $\hat{\rho} \longrightarrow \rho(x, x') = \langle x | \hat{\rho} | x' \rangle$

$$\hat{\rho}_{\psi} = |\psi\rangle\langle\psi|$$

$$\hat{\rho} = \sum_n P_n |\psi_n\rangle\langle\psi_n| , \quad P_n \geq 0, \ \sum_n P_n = 1$$

$$|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} ; \quad \langle\psi| = (1 \ \ 0)$$

$$|\psi\rangle\langle\psi| = \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1 \ \ 0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \equiv \hat{P}_1$$



$$\hat{P}_1 \vec{a} = \vec{a}_1$$

$$\psi_1(x)\ , \quad \psi_2(x)$$

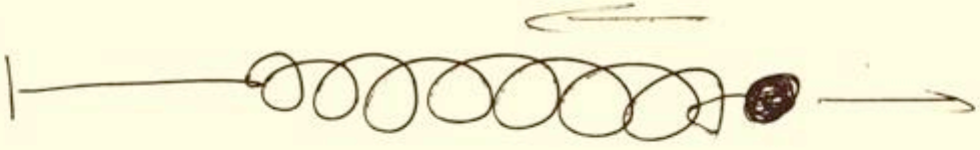
$$\Psi(x) = c_1\,\psi_1(x) + c_2\,\psi_2(x)$$

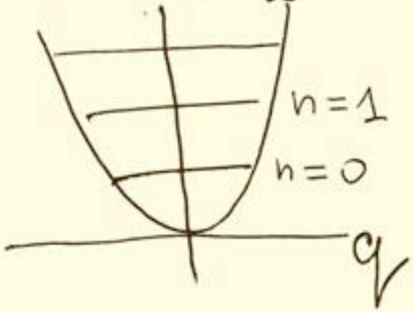
$$|\psi\rangle = \sqrt{p_1}\,|\psi_1\rangle + \sqrt{p_2}\,e^{i\varphi}\,|\psi_2\rangle$$

$$\hat{\rho}_\psi = p_1\,\hat{\rho}_1 + p_2\,\hat{\rho}_2 +$$

$$+ \sqrt{p_1 p_2}\;\frac{\hat{\rho}_1\hat{P}_0\hat{\rho}_2 + \hat{\rho}_2\hat{P}_0\hat{\rho}_1}{\sqrt{\mathrm{Tr}\,\hat{\rho}_1\hat{P}_0\hat{\rho}_2\hat{P}_0}}$$

$V(q) = \frac{q^2}{2}$



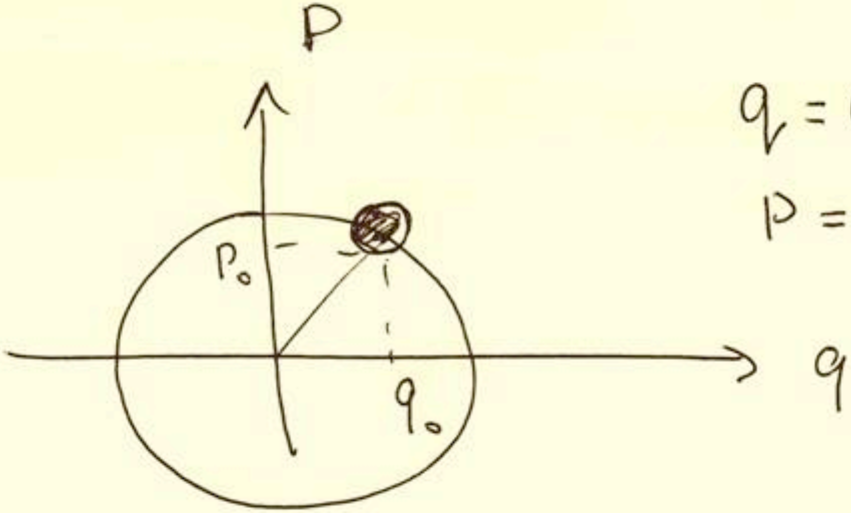


$$\Psi_\alpha(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt[4]{\pi}}\, e^{-\frac{|\alpha|^2}{2} - \frac{\alpha^2}{2} + \sqrt{2}\,\alpha x}$$

$$|\Psi_\alpha(x)|^2 = \frac{1}{\sqrt{2\pi\sigma_x^2}} \exp -\frac{(x-\bar{x})^2}{2\sigma_x^2}$$

$$\sigma_x^2 = \frac{1}{2}, \qquad \bar{x} = \sqrt{2}\,\mathrm{Re}\,\alpha, \quad \bar{p} = \sqrt{2}\,\mathrm{Im}\,\alpha$$

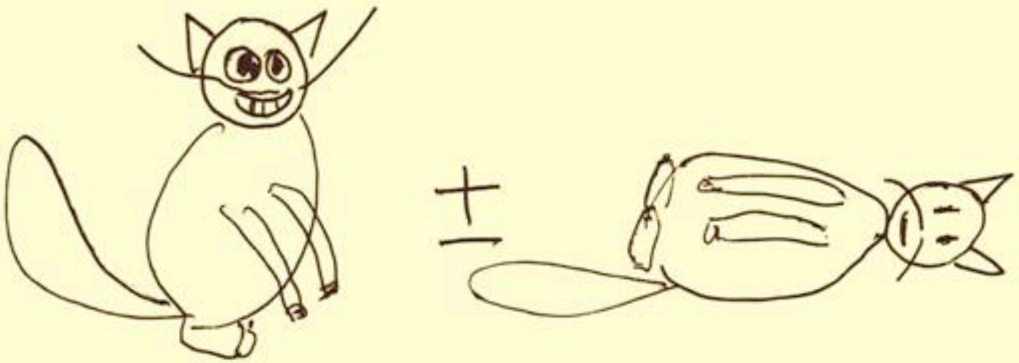
$$\sigma_p^2 = \frac{1}{2}; \qquad \sigma_x^2\sigma_p^2 = \frac{1}{4}.$$



$$q = q_0 \cos t + p_0 \sin t$$
$$p = p_0 \cos t - q_0 \sin t$$

$$\hat{p}_0(t) = \hat{p} \cos t + \hat{q} \sin t \quad ; \quad \hat{q} = x$$
$$\hat{q}_0(t) = +\hat{q} \cos t - \hat{p} \sin t \qquad \hat{p} = -i \frac{\partial}{\partial x}$$

$$\frac{d}{dt} \hat{p}_0(t) = 0, \quad \frac{d}{dt} \hat{q}_0(t) = 0 .$$



(1935)

(1974)

$$\Psi_{\alpha_+}(x) = N_+ \left(\Psi_\alpha(x) + \Psi_{-\alpha}(x) \right)$$

$$\Psi_{\alpha_-}(x) = N_- \left(\Psi_\alpha(x) - \Psi_{-\alpha}(x) \right)$$

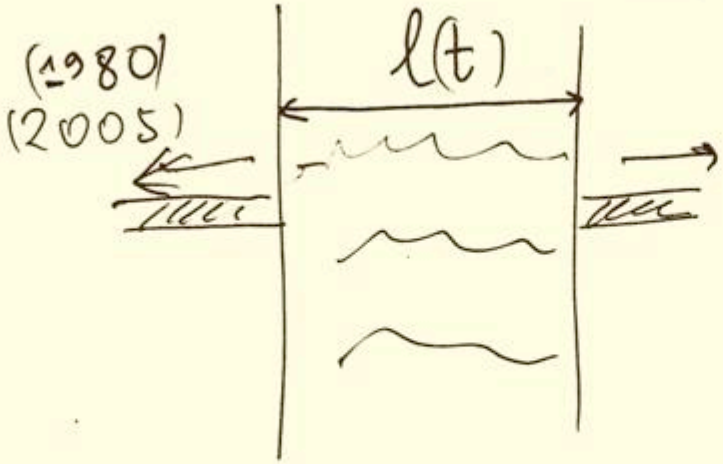
$$\hat{H} = \frac{\hat{p}^2}{2} + \frac{\omega^2(t)\hat{q}^2}{2}, \quad \omega(0)=1$$

(1969)

$$\hat{A}(t) = \frac{i}{\sqrt{2}}\left(\varepsilon(t)\hat{p} - \dot{\varepsilon}(t)\hat{q}\right)$$

$$\frac{d\hat{A}(t)}{dt} = 0\,; \qquad \ddot{\varepsilon} + \omega^2(t)\varepsilon = 0, \quad \varepsilon(0)=1, \ \dot{\varepsilon}(0)=i$$

$$\Psi_0(x,t) = \frac{1}{\sqrt[4]{\pi}\,\sqrt{\varepsilon(t)}} \exp\left[\frac{i\,\dot{\varepsilon}(t)\,x^2}{2\,\varepsilon(t)}\right]$$



$$E_v \approx \sum_k{}' \frac{\hbar\omega_k(l)}{2}$$

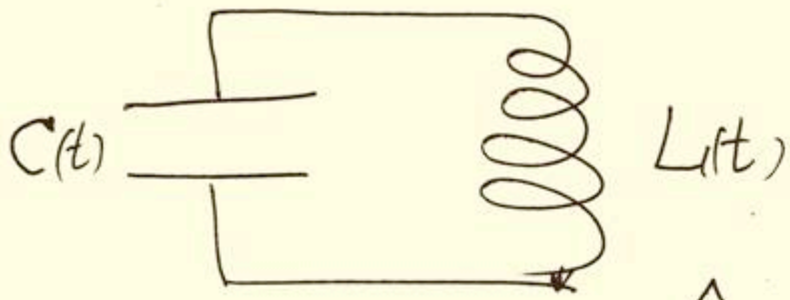
$$F_c \sim \frac{\partial E_v}{\partial l}$$

$$\sigma_x^2 \sigma_p^2 \geqslant \frac{\hbar^2}{4} \frac{1}{1-r^2}$$

$$\sigma_x^2 = \frac{|\varepsilon(t)|^2}{2} \longleftarrow (\hbar = 1)$$

$$\sigma_p^2 = \frac{|\dot{\varepsilon}(t)|^2}{2} \leftarrow (\hbar = 1)$$

$$\sigma_x^2 \sigma_p^2 = \frac{1}{4} \frac{1}{1-r^2}$$



$$\omega \sim \frac{1}{\sqrt{LC}}$$

$$\hat{q} \sim \hat{V}$$

$$\hat{p} \sim \hat{I}$$

$$\frac{\hbar\omega}{T} \gg 1$$

(1980)

(2008)

$$\Psi_s(x_1, x_2) = \Phi(x_1)\,\chi(x_2)$$

$$\Psi_e(x_1, x_2) = c_1 \Phi_1(x_1)\,\chi_1(x_2) + c_2 \Phi_2(x_1)\,\chi_2(x_2)$$

$s_1 = 1/2$

$s_2 = 1/2$

$s_1 \oplus s_2 = (0, 1)$

$$\to \left| s = 1, s_z = 0 \right\rangle = \frac{1}{\sqrt{2}} \left\{ |\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2 \right\}$$

$$\rho = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \rho = \rho^+, \; \mathrm{Tr}\,\rho = 1$$

$$\rho \geq 0$$

$$\hat{\rho}_{ss}(1,2) = \hat{\rho}_1(1) \otimes \hat{\rho}_2(2)$$

$$\hat{\rho}_s(1,2) = \sum_n p_n \, \hat{\rho}_1^{(n)}(1) \otimes \hat{\rho}_2^{(n)}(2)$$

$$p_n \geq 0 \, , \quad \sum_n p_n = 1$$

$$\hat{\rho}_e(1,2) \neq \sum_n p_n \, \hat{\rho}_1^{(n)}(1) \otimes \hat{\rho}_2^{(n)}(2)$$

$$t \longrightarrow -t \; ; \; \Psi \longrightarrow \Psi^*$$

$$\Psi(x)\,\Psi^*(x') \longrightarrow \Psi^*(x)\,\Psi(x')$$

$$\rho_\Psi \longrightarrow \rho_\Psi^{tr}, \quad \mathrm{Tr}\,\rho^{tr} = \mathrm{Tr}\,\rho = 1, \quad \rho^{tr} \geq 0$$

$$\hat{\rho}_s^{ppt}(1,2) = \sum_n p_n\, \hat{\rho}_1^{(n)}(1) \otimes \hat{\rho}_2^{(n)\,tr}(2)$$

$$\left(\hat{\rho}_s^{ppt}(1,2)\right)^+ = \hat{\rho}_s^{ppt}(1,2) \geq 0$$

$$\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix} \otimes \begin{pmatrix} A & B \\ C & D \end{pmatrix}\right]^{ppt} =$$

$$= \begin{pmatrix} a\begin{pmatrix} A & B \\ C & D \end{pmatrix} & b\begin{pmatrix} A & B \\ C & D \end{pmatrix} \\ c\begin{pmatrix} A & B \\ C & D \end{pmatrix} & d\begin{pmatrix} A & B \\ C & D \end{pmatrix} \end{pmatrix}$$

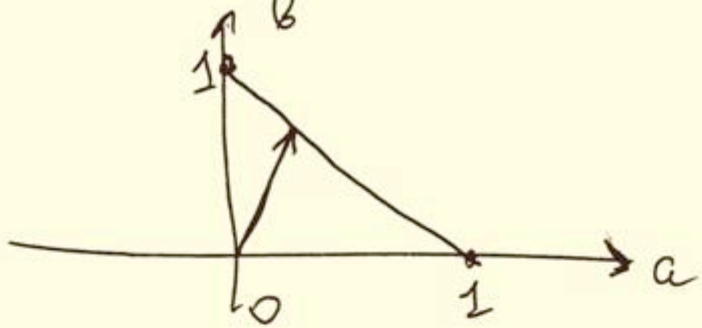
$$\rho(1,2)^{ppt} = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^{ppt} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_1 = \frac{1}{2}1, \quad \lambda_2 = \frac{1}{2}1, \quad \lambda_3 = \frac{1}{2}1, \quad \boxed{\lambda_4 = -\frac{1}{2}} \; !!$$

$$1 \geq x, y, z, t \geq 0$$

$$B_1 = \begin{pmatrix} x & y \\ 1-x & 1-y \end{pmatrix}, \quad B_2 = \begin{pmatrix} z & t \\ 1-z & 1-t \end{pmatrix}$$



$m_{1,2} = \pm 1$ $\langle m_x \rangle = 2x-1$, $\langle m_y \rangle = 2y-1$ $\langle m_z \rangle = 2z-1$, $\langle m_t \rangle =$
$= 2t-1$

$$B_{ss} = B_1 \otimes B_2$$

$$I = \begin{pmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{pmatrix}$$

$$B_{ss} = \begin{pmatrix} xz & xt & yz & yt \\ x(1-z) & x(1-t) & y(1-z) & y(1-t) \\ (1-x)z & (1-x)t & (1-y)z & (1-y)t \\ (1-x)(1-z) & (1-x)(1-t) & (1-y)(1-z) & (1-y)(1-t) \end{pmatrix}$$

$$\langle m_x m_z \rangle = 4xz - 2x - 2z + 1$$

$$\mathrm{Tr}\left(\underset{\approx}{I}\,\underset{\approx}{B}\right) = \langle m_x m_z \rangle + \langle m_x m_t \rangle + \langle m_y m_z \rangle - \langle m_y m_t \rangle = \tilde{B}(x, y, z, t)$$

$$= 4\,(zx + xt + yz - yt) - 4x - 4z + 2$$

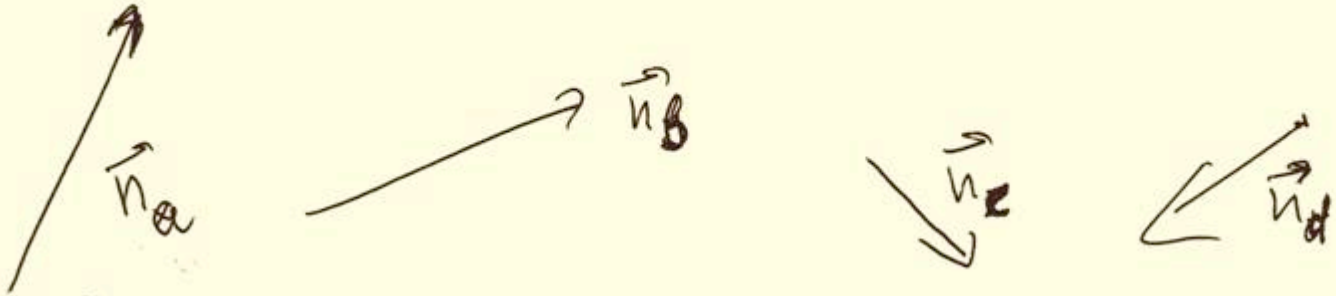
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial t^2}\right)\tilde{B}(x, y, z, t) = 0$$

$$\max|\tilde{B}| = 2$$

$$\boxed{|\tilde{B}| \leq 2}$$

$$B_S = \sum_n{}' p_n \, B_1^{(n)} \otimes B_2^{(n)}$$

$$|\tilde{B}| = |Tr \, I \, B_s| \leq 2$$



$$\langle \vec{\sigma}_1 \vec{n}_a \, \vec{\sigma}_2 \vec{n}_b \rangle ; \quad \langle \vec{\sigma}_1 \vec{n}_a \, \vec{\sigma}_2 \vec{n}_c \rangle ; \quad \langle \vec{\sigma}_1 \vec{n}_d \, \vec{\sigma}_2 \vec{n}_b \rangle , \quad \langle \vec{\sigma}_1 \vec{n}_d \, \vec{\sigma}_2 \vec{n}_c \rangle$$

$$\max |\tilde{B}| = 2\sqrt{2} > 2 \; ; \quad \langle \bullet \rangle = Tr(\rho(1,2), \bullet)$$

1932

$$W(q,p) = \int \Psi\left(q+\frac{u}{2}\right) \Psi^*\left(q-\frac{u}{2}\right) e^{-ipu}\, du$$

$$W^* = W \quad ; \quad \int W(q,p) \frac{dq\,dp}{2\pi} = 1$$

$$\int W(q,p) \frac{dp}{2\pi} = |\Psi(q)|^2 = |\langle q|\Psi\rangle|^2$$

$$\int W(q,p) \frac{dq}{2\pi} = |\tilde{\Psi}(p)|^2 = |\langle p|\Psi\rangle|^2$$

$$\frac{W(q,p)}{2\pi} \xrightarrow{\quad ? \quad} f(q,p)$$

$$\langle \hat{q}^n \rangle = \mathrm{Tr}\, \hat{\rho}\, \hat{q}^n \equiv \int q^n \frac{W(q,p)}{2\pi}\, dq\, dp$$

$$\langle \hat{p}^n \rangle = \mathrm{Tr}\, \hat{\rho}\, \hat{p}^n = \int p^n \frac{W(q,p)}{2\pi}\, dq\, dp$$

$$W(q,p) \gtrless 0$$

$$\rho(x, x') = \frac{1}{2\pi} \int W\left(\frac{x+x'}{2}, p\right) e^{ip(x-x')}\, dp$$

$$\hat{\rho} \longleftrightarrow W$$